BLIND SOURCES SEPARATION USING BILINEAR AND QUADRATIC TIME-FREQUENCY REPRESENTATIONS

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ABSTRACT

We consider here the problem of blind sources separation. During the last decade, many solutions have been proposed among which contrasts functions, maximum likelihood functions, information-theoretic criteria, etc... More recently, a new method based on some time-frequency (*t-f*) representations has been introduced by Belouchrani *et al*. It consists in joint-diagonalizing a combined set of "spatial *t-f* distributions (*stfd*)" matrices. However, *t-f* representations properties still not have been widely exploited to solve the sources separation problem. Our aim is to develop this point to take better advantage of bilinear and quadratic time-frequency representations properties. Hence, we derive new criteria of choice of *stfd* matrices sets to be joint-diagonalized and/or joint anti-diagonalized. Finally, some computer simulations are presented in order to demonstrate the effectiveness of the proposed algorithm.

1. INTRODUCTION

In the past ten years, many "blindly" operating approaches have dealt with a model commonly known as *sources separation*. In such a problem, the coupling channels are assumed to have unknown constant gains. The goal is then to recover the inputs from the only outputs, without the explicit use of the unobservable sources assumed independent. Many solutions have been proposed to solve this problem among which contrasts functions [7], maximum likelihood functions or information-theoretic criteria [4]... In this communication we focus on a method recently introduced in [2], based on the joint-diagonalization of a combined set of "spatial *t-f* distributions" matrices (joint- diagonalization methods being classically used in sources separation [3] [2] [6]). Yet, *t-f* representations properties still not have been widely exploited to solve the sources separation problem. That is the main goal of this paper.

After recalling some of the properties of the bilinear and quadratic time-frequency representations we use, we explain how it is possible to take advantage of these properties to derive new criteria of choice of *stfd* matrices sets to be joint-diagonalized and/or joint anti-diagonalized for sources separation. Finally, some computer simulations are presented in order to demonstrate the effectiveness of the algorithm and a discussion on which kind of quadratic *t-f* representation to choose is also proposed.

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2. PROBLEM STATEMENT

2.1. Model and assumptions

We consider the blind sources separation problem where n sources signals are received on m sensors (assuming that $m \geq n$). In matrix and vector notations, the input/output relationship of the mixing system reads:

$$\boldsymbol{x}(t) = \boldsymbol{y}(t) + \boldsymbol{b}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{b}(t) \tag{1}$$

with $\boldsymbol{A}$ the $m \times n$ instantaneous mixing matrix, $\boldsymbol{x}(t) = [x_1(t), ..., x_m(t)]^T$ the $m \times 1$ observations vector (superscript ^{T} denoting transposition), $\boldsymbol{s}(t) = [s_1(t), ..., s_n(t)]^T$ the $n \times 1$ sources vector and $\boldsymbol{b}(t) = [b_1(t), ..., b_m(t)]^T$ the additive noise.

The noise is assumed to be stationary, zero mean, with components independent of the sources and mutually independent ; the correlation matrix of the noise is then:

$$\mathbf{R}_{\mathbf{b}}(\tau) = \mathsf{E}[\boldsymbol{b}(t+\tau)\boldsymbol{b}^*(t)] = \sigma^2 \delta(\tau)\mathbf{I}_{\mathsf{n}} \tag{2}$$

where $\mathbf{I}_{\mathsf{n}}$ is the $n \times n$ identity matrix, $\delta(\cdot)$ the dirac distribution and $\mathsf{E}[\cdot]$ the mathematical expectation operator.

In the following, we limit ourself to the case of a real mixing matrix and real sources. The random sources are assumed non-stationary and independent having as correlation matrix:

$$\mathbf{R}_{\mathbf{s}}(\tau) = \mathsf{diag}[r_{11}(\tau), \ldots, r_{nn}(\tau)] \tag{3}$$

with $\mathsf{diag}[.]$ standing for a diagonal matrix. These sources are supposed to have structures such as their realizations have different localization properties in the *t-f* plane.

The problem of blind sources separation is then to identify the mixing matrix in order to restore the source signals.

2.2. Problem indeterminacies

It is well known that the sources separation problem can be solved only up to a diagonal matrix $\boldsymbol{D}$ (which corresponds to arbitrary attenuations for the restored sources), and a permutation matrix $\boldsymbol{P}$ (which corresponds to an arbitrary order of restitution). That is why the unit power assumption on sources can be done without loss of generality ; it finally leads to: $\mathbf{R_s}(\tau) = \mathbf{I_n}$ involving that $\mathbf{R_y} = \boldsymbol{A}\boldsymbol{A}^T$.

3. BILINEAR & QUADRATIC TIME-FREQUENCY REPRESENTATIONS

3.1. Some recalls

3.1.1. Bilinear and Quadratic TFR

A Quadratic Time-Frequency Representation (*Q-TFR*) associated to a signal $x_i(t)$, is [5] the restriction to $x_i(t) = x_j(t)$ of a bilinear transform applied to a couple (x_i, x_j) :

$$x_i \overset{Q-TFR}{\longleftrightarrow} D_{x_i} = D_{x_i x_i} \text{ such as } (x_i, x_j) \overset{B-TFR}{\longleftrightarrow} D_{x_i x_j} \quad (4)$$

$$D_{x_i x_j}(t, \nu; R) = \int_{\mathsf{R}^2} x_i(\theta) x_j^*(\theta') \underbrace{R(\theta, \theta'; t, \nu)}_{kernel} d\theta d\theta'. \quad (5)$$

To simplify, we will omit the dependence on the kernel R in the notations of $D_{x_i x_j}$ and D_{x_i}.

A *Q-TFR*, $D_x(t, \nu)$, associated to $x(t) \overset{\mathbf{F}}{\leftrightarrow} X(\nu)$, is said **energetic** if, $\forall x$, it satisfies:
$\int_{\mathsf{R}^2} D_x(t, \nu) dt d\nu = E_x = \int_{\mathsf{R}} |x(t)|^2 dt = \int_{\mathsf{R}} |X(\nu)|^2 d\nu$ with E_x the energy of the signal and $X(\nu)$ its Fourier transform.

3.1.2. Cohen's & Affine classes

The Cohen's class is the class of energetic *TFRs* covariant under time and frequency shifts whereas the Affine class is the class of energetic *TFRs* covariant under time scalings and shifts. Their members, associated to $x(t)$ and noted ψ_x and ϖ_x, satisfy the condition of definition of the class and can be written in a general form characterizing the class. They also conform to the energetic condition involving specific properties of the generating kernels (cf. Table 1). By playing upon the kernel it is then possible to generate the different elements of a given class (cf. Table 2). Finally, it is possible to give some exemples of representations belonging to the Cohen's class: Spectrogram (Sp), Pseudo Wigner-Ville (PWV), Smoothed Pseudo Wigner-Ville (SPWV), etc..., to the Affine Class: Scalogramme (Sc) and to both classes: Wigner-Ville (WV), Choï-Williams (CW) (see [5] for more details about these representations).

Table 1: A summary of Affine and Cohen's class properties & definitions

Cohen class condition of definition: $x(t) \leftrightarrow \psi_x(t, \nu) \quad \Rightarrow$ $y(t) = x(t-\theta) e^{2\mathbf{i}\pi\eta t} \quad \leftrightarrow \quad \psi_y(t, \nu) = \psi_x(t-\theta, \nu-\eta)$ Affine class condition of definition: $x(t) \leftrightarrow \varpi_x(t, \nu) \quad \Rightarrow$ $y(t) = \lvert a\rvert^{\frac{1}{2}} x(a(t-\tau)) \quad \leftrightarrow \quad \varpi_y(t, \nu) = \varpi_x(a(t-\tau), \frac{\nu}{a})$
General form of Cohen's class *TFR*: $\psi_{xy}(t, \nu) = \iint x(\theta + \frac{\tau}{2}) y^*(\theta - \frac{\tau}{2}) K(t-\theta, \tau) e^{-2\mathbf{i}\pi\nu\tau} d\tau d\theta$ $\psi_x(t, \nu) = \psi_{xx}(t, \nu)$ General form of Affine Class *TFR*: $\varpi_{xy}(t, \nu) = \iint x(\theta + \frac{\tau}{2}) y^*(\theta - \frac{\tau}{2}) \lvert\nu\rvert K'(\nu(t-\theta), -\nu\tau) d\tau d\theta$ $\varpi_x(t, \nu) = \varpi_{xx}(t, \nu)$
Energetic condition satisfied by the kernels of: Cohen's class: $\int K(t, 0) dt = 1$. Affine class: $\iiint \frac{1}{\lvert\nu\rvert} K'(t, \tau) e^{-2\mathbf{i}\pi\nu\tau} dt d\tau d\nu = 1$.

Table 2: Kernels of some Q-TFR.

Transf.	Kernel $K(t, \tau)$ in Cohen class	Kernel $K'(t, \tau)$ in Affine class
WV	$\delta(t)$	$\delta(t) e^{2\mathbf{i}\pi\tau}$
PWV	$\delta(t) H(\tau)$	/
SPWV	$G(t) H(\tau)$	/
Sp	$H(-\frac{\tau}{2} - t) H^*(\frac{\tau}{2} - t)$	/
Sc	/	$\lvert\frac{1}{\nu_0}\rvert H_0^*(\frac{-t-\frac{\tau}{2}}{\nu_0}) H_0(\frac{-t+\frac{\tau}{2}}{\nu_0})$
CW	$\sqrt{\frac{\sigma}{4\pi}} \frac{1}{\lvert\tau\rvert} e^{-\frac{\sigma t^2}{4\tau^2}}$	$\sqrt{\frac{\sigma}{4\pi}} \frac{1}{\lvert\tau\rvert} e^{-\frac{\sigma t^2}{4\tau^2}} e^{2\pi\mathbf{i}\tau}$

3.2. Spatial *B-TFR*

In the case of a vectorial signal $\boldsymbol{x}(t) = [x_1(t), \ldots, x_m(t)]^T$, the bilinear transform is spatial:

$$\mathbf{D_{xx}}(t, \nu) = \int_{\mathsf{R}^2} \boldsymbol{x}(\theta) \boldsymbol{x}^H(\theta') R(\theta, \theta'; t, \nu) d\theta d\theta' \quad (6)$$

or equivalently:

$$\mathbf{D_{xx}}(t, \nu) = \begin{pmatrix} D_{x_1 x_1}(t, \nu) & \ldots & D_{x_1 x_m}(t, \nu) \\ \vdots & \ddots & \vdots \\ D_{x_m x_1}(t, \nu) & \ldots & D_{x_m x_m}(t, \nu) \end{pmatrix}. \quad (7)$$

The terms on the diagonal are called auto-terms whereas the other are called inter-terms.

3.3. Some useful properties of *B*- and *Q-TFR*

For source separation, the following two properties are of particular interest.

Property 1: Hermitian symetry of *B-TFR* **-** A bilinear time-frequency representation exhibits hermitian symetry if and only if it satisfies:

$$D_{x_i x_j}(t,\nu) = D^*_{x_j x_i}(t,\nu) \tag{8}$$

which for the kernel, leads to $R(\theta,\theta';t,\nu) = R^*(\theta',\theta;t,\nu)$. This property also involves conditions for the kernels of Cohen's class:

$$K(t-\theta,\tau) = K^*(t-\theta,-\tau) \tag{9}$$

Affine class:

$$K'(t-\theta,-\nu\tau) = K'^*(t-\theta,\nu\tau). \tag{10}$$

One can check that these properties are satisfied by many transformations among which Wigner-Ville, spectrogram, scalogram and Choï-Williams distribution. With regard to smoothed versions of Wigner-Ville, the smoothing window also has to present hermitian symmetry.

Property 2: Reality of *Q-TFR* **-** A quadratic time-frequency representation is real if and only if it satisfies:

$$D_{x_i x_i}(t,\nu) = D^*_{x_i x_i}(t,\nu) \tag{11}$$

On can check that this property is satisfied by many *Q-TFR* among which Wigner-Ville and its smoothed versions, the spectrogram, scalogram and the Choï-Williams distribution.

4. BLIND SOURCES SEPARATION USING BILINEAR TFR

4.1. First stage: spatial whitening

The mixing matrix $\boldsymbol{A}$ can be parameterized as $\boldsymbol{V}\boldsymbol{\Delta}^{\frac{1}{2}}\boldsymbol{U}$ with $\boldsymbol{V}$ and $\boldsymbol{U}$ unitary matrices and $\boldsymbol{\Delta}$ a diagonal matrix [8]. The correlation matrix of the observed signals at $\tau = 0$ is given by: $\mathbf{R_x}(0) = \boldsymbol{A}\mathbf{R_s}(0)\boldsymbol{A}^T + \mathbf{R_b}(0) = \boldsymbol{A}\boldsymbol{A}^T + \sigma^2\mathbf{I}_n$
$\Rightarrow \boldsymbol{A}\boldsymbol{A}^T = \boldsymbol{V}\boldsymbol{\Delta}\boldsymbol{V}^T = \mathbf{R_x}(0) - \sigma^2\mathbf{I}_n$.
Moreover, the eigenvalues decomposition of $\mathbf{R_x}(0)$, which is unique, writes: $\mathbf{R_x}(0) = \boldsymbol{Q}\boldsymbol{\Sigma}\boldsymbol{Q}^T$ with $\boldsymbol{Q}$ a unitary matrix and $\boldsymbol{\Sigma}$ a diagonal matrix. By identification, it is found that: $\boldsymbol{V} = \boldsymbol{Q}$ and $\boldsymbol{\Delta} = \boldsymbol{\Sigma} - \sigma^2\mathbf{I}_n$. The spatial whitening matrix $\boldsymbol{W}$ is then defined as: $\boldsymbol{W} = \boldsymbol{\Delta}^{-\frac{1}{2}}\boldsymbol{V}^T$ and the whitened signals are defined as:

$$\boldsymbol{z}(t) = \boldsymbol{W}\boldsymbol{x}(t) = \boldsymbol{W}\boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{W}\boldsymbol{n}(t) = \boldsymbol{U}\boldsymbol{s}(t) + \boldsymbol{W}\boldsymbol{n}(t) \tag{12}$$

In the noiseless case, it simply gives: $\boldsymbol{z}(t) = \boldsymbol{U}\boldsymbol{s}(t)$ which leads to $\mathbf{R_z}(\tau) = \boldsymbol{U}\mathbf{R_s}(\tau)\boldsymbol{U}^T = \mathbf{I}_\mathsf{n}$. The problem has been reduced to the case of a unitary mixing of signals: the unitary matrix $\boldsymbol{U}$ still has to be estimated to be able to perform separation.

4.2. Second stage: use of spatial bilinear time-frequency distributions

4.2.1. Spatial bilinear t-f distribution of a linear mixtures of sources

In the noiseless case, the spatial quadratic *t-f* distribution of the whitened signals reads:

$$\begin{aligned}\mathbf{D_{zz}}(t,\nu) = \boldsymbol{W}\mathbf{D_{xx}}(t,\nu)\boldsymbol{W}^T &= \boldsymbol{W}\boldsymbol{A}\mathbf{D_{ss}}(t,\nu)\boldsymbol{A}^T\boldsymbol{W}^T \\ &= \boldsymbol{U}\mathbf{D_{ss}}(t,\nu)\boldsymbol{U}^T \end{aligned}\tag{13}$$

whose (k,p)-th element is: $D_{z_k z_p}(t,\nu) = \sum_{i,j=1}^n u_{ki}u_{pj} D_{s_i s_j}(t,\nu)$.
In the following the time t and the frequency ν will be omitted to simplify the notations.

• Case $k = p$ (auto-terms of spatial *t-f* distribution):

$$D_{z_k z_k} = \sum_{i,j=1}^n u_{ki}u_{kj}D_{s_i s_j} \tag{14}$$

$$= \underbrace{\sum_{i=1}^n u_{ki}u_{ki}D_{s_i s_i}}_{T} + \underbrace{\sum_{i,j(j\neq i)=1}^n u_{ki}u_{kj}D_{s_i s_j}}_{V} \tag{15}$$

Using Property 2 and the fact that $u_{ki}u_{ki}$ is real, T is found to be real. Now, we calculate V:

$$\begin{aligned} V &= \sum_{i,j(i<j)=1}^n u_{ki}u_{kj}D_{s_i s_j} + \sum_{i,j(i>j)=1}^n u_{ki}u_{kj}D_{s_i s_j} \\ &= \sum_{i,j(i<j)=1}^n u_{ki}u_{kj}D_{s_i s_j} + \sum_{i,j(i<j)=1}^n u_{kj}u_{ki}D_{s_j s_i} \\ &= \sum_{i,j(i<j)=1}^n (u_{ki}u_{kj}D_{s_i s_j} + u_{ki}u_{kj}D_{s_j s_i}) \end{aligned}$$

Introducing Property 1, it is found that:

$$\begin{aligned} V &= \sum_{i,j(i<j)=1}^n u_{ki}u_{kj}(D_{s_i s_j} + D^*_{s_i s_j}) \\ &= \sum_{i,j(i<j)=1}^n u_{ki}u_{kj}(D_{s_i s_j} + D_{s_i s_j})^*) \\ &= 2\sum_{i,j(i<j)=1}^n u_{ki}u_{kj}\mathsf{Real}\{D_{s_i s_j}\} \end{aligned}$$

As a consequence V and finally the auto-terms $D_{z_k z_k}$ are real.

• Case k≠ p (inter-terms of the *t-f* distribution):

$$\begin{aligned} D_{z_k z_p} &= \sum_{i,j=1}^{n} u_{ki} u_{pj} D_{s_i s_j} \\ &= \sum_{i=1}^{n} u_{ki} u_{pi} D_{s_i s_i} + \sum_{i,j(i \neq j)=1}^{n} u_{ki} u_{pj} D_{s_i s_j} \end{aligned}$$

$u_{ki}u_{pi}$ is real, so the first term too. With regard to the second term, it is easy to show that it is generally complex because $D_{s_i s_j}$ is complex.

It implies that the imaginary part of the spatial *t-f* representation only corresponds to the interferences between sources signals whatever the considered point in the time-frequency plane. As consequence, we propose to use this result to decide what points in the time-frequency plane correspond to signals and what points correspond to interferences.

Moreover, this approach can be generalized to the case of complex sources or complex mixing matrices, by treating the real part and the imaginary part independently of each other.

4.2.2. New diagonalization & anti-diagonalization separation criteria

As we have:
$D_{z_k z_p}(t,\nu)$

$$= \sum_{i=1}^{n} u_{ki} u_{pi} D_{s_i s_i}(t,\nu) + \sum_{i,j(i \neq j)=1}^{n} u_{ki} u_{pj} D_{s_i s_j}(t,\nu) \tag{16}$$

different cases have to be considered, depending on the time-frequency point (t_0, ν_0) considered in the time-frequency plane:

- When at least one signal is present, without any interference between sources (the imaginary part of the *stfd* is null whereas its real part is different from 0 in such a *t-f* point) then the set $\{\mathbf{D_{zz}}(t_0,\nu_0), \exists \, \{i\} \neq \{\emptyset\} \, / D_{s_i s_i}(t_0,\nu_0) \neq 0, \, D_{s_k s_j}(t_0,\nu_0) = 0, \forall \, (k,j), \, k \neq j\}$ has to be **joint-diagonalized**. The spatial *t-f* distribution of the whitened signals writes:

  $\mathbf{D_{zz}}(t_0,\nu_0) =$

  $$\begin{pmatrix} \sum_i u_{1i} u_{1i} D_{s_i s_i} & \dots & \sum_i u_{1i} u_{ni} D_{s_i s_i} \\ \vdots & \ddots & \vdots \\ \sum_i u_{ni} u_{1i} D_{s_i s_i} & \dots & \sum_i u_{ni} u_{ni} D_{s_i s_i} \end{pmatrix} \tag{17}$$

  and it is real.

- When there is no signal, but only interferences (the *stfd* is complex in such a *t-f* point) then the set $\{\mathbf{D_{zz}}(t_0,\nu_0), \forall i, D_{s_i s_i}(t_0,\nu_0) = 0, \exists \{(i,j); i \neq j\} \neq \{\emptyset\} / D_{s_i s_j}(t_0,\nu_0) \neq 0\}$, has to be joint anti-diagonalized. The diagonal terms of the *stfd* $\mathbf{D_{zz}}(t_0,\nu_0)$ of the whitened signals have the following form:
  $2 \sum_{i,j(i<j)} u_{ki} u_{kj} \mathsf{Real}\{D_{s_i s_j}\}$
  whereas the other terms have the following form:
  $\sum_{i,j(i \neq j)} u_{ki} u_{pj} D_{s_i s_j}$. Terms on the diagonal of the matrix are real, whereas the other ones are complex.

- When both signals and interferences are present or absent in (t_0, ν_0), it is better not to take this *t-f* point into account. The estimation of the unitary matrix would be degraded.

5. COMPUTER SIMULATIONS

In this experiment, we consider three real modulations of frequency: two linear ones and a sinusoïdal one. The mixing matrix is:

$$A = \begin{pmatrix} 1 & 0.8 & 0.9 \\ 0.7 & 1 & 0.85 \\ 1.3 & 0.5 & 1 \end{pmatrix}$$

The number of points is 256 and the number of frequency bins used in the *t-f* representations is 64. The plots of the three individual modulations are shown on Fig. 1. On Fig. 2, the mixed signals are displayed. The Wigner-Ville distributions of the mixtures are depicted in Fig. 3 and 4 (real and imaginary part resp.). On Fig. 5 and 6 are displayed the *t-f* points retained by the new criteria of decision between signals and interferences. Finally, Fig. 7, 8, 9 and 10 represent the estimated signals thanks to joint-diagonalization (Fig. 7) and joint anti-diagonalization (Fig. 8). The sources are well estimated as it appears on the spatial Wigner-Ville distributions of the signals estimated by joint anti-diagonalization which are depicted in Fig. 9 (real part) and 10 (imaginary part).

6. CONCLUSION

As a conclusion, in this communication, we have focused on blind sources separation based on spatial bilinear time-frequency representations. New criteria of choice between signals and interferences have been introduced, determining which matrices set has to be joint-diagonalized and which matrices set has to be anti joint-diagonalized. These criteria have been experimented on real non stationary signals in order to illustrate their effectiveness. Concerning the choice of the bilinear *TFRs* to use with such criteria: we have given the conditions that the *TFRs* have to satisfy. It also important that the used *TFR* presents a lot of interferences between sources. That is why we have used the Wigner-Ville distribution rather than the spectrogram for example...

7. REFERENCES

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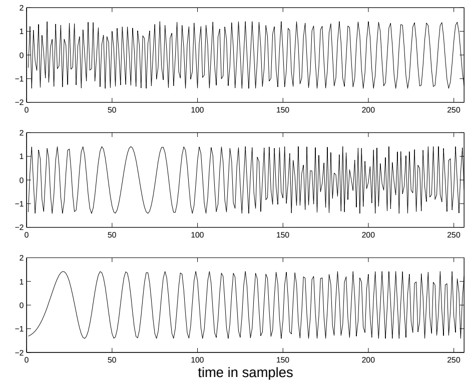

Fig. 1. The three sources

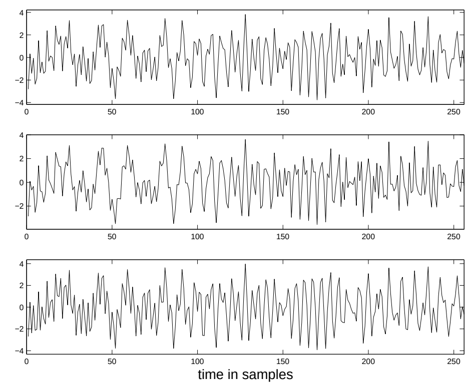

Fig. 2. The three mixed signals

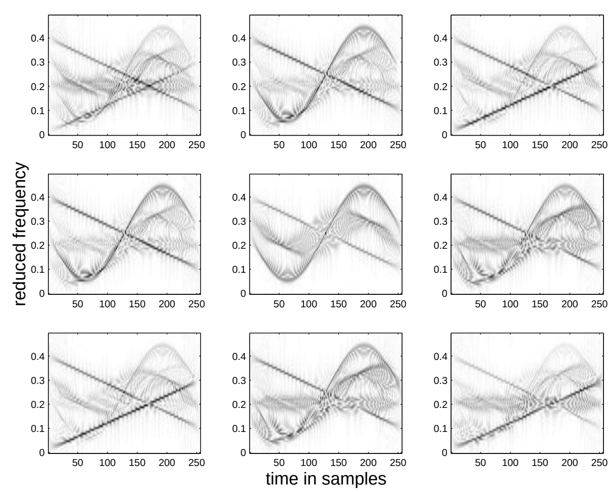

Fig. 3. Real part of the spatial Wigner-Ville distribution of the whitened mixture

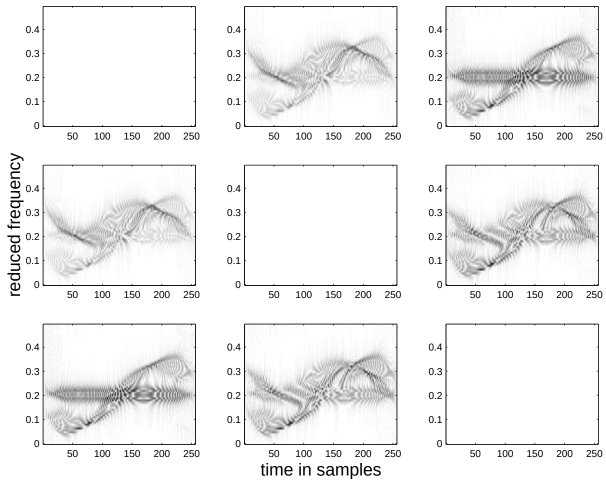

Fig. 4. Imaginary part of the spatial Wigner-Viller distribution of the whitened mixture

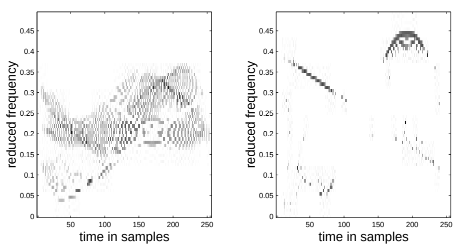

Fig. 5. Points in the *t*-*f* plane kept for joint anti-diagonalization (left) and joint diagonalization (right)

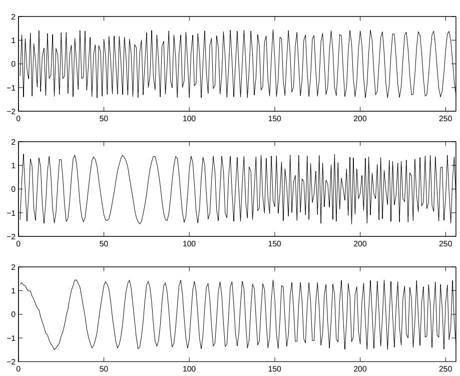

Fig. 6. Sources estimated by joint anti-diagonalization

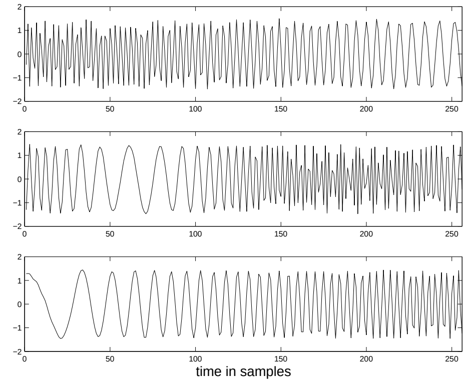

Fig. 7. Sources estimated by joint-diagonalization

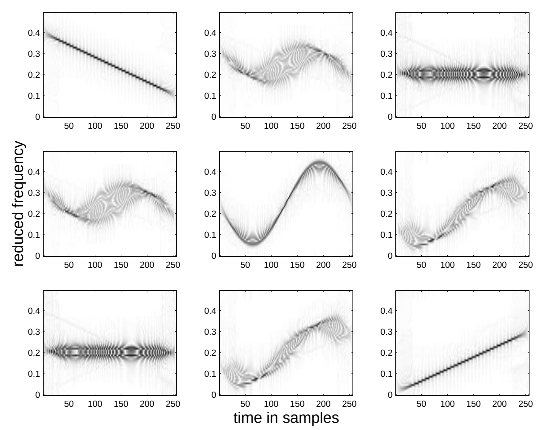

Fig. 8. Real part of the spatial Wigner-Ville distribution of the signals estimated by joint anti-diagonalization

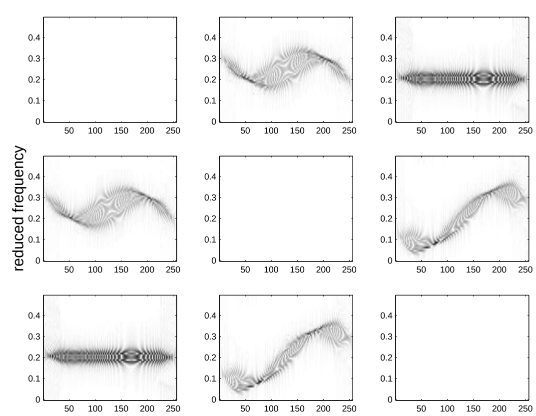

Fig. 9. Imaginary part of the spatial Wigner-Ville distribution of the signals estimated by joint anti-diagonalization