MULTIPLICATIVE NOISE REMOVAL USING SELF-ORGANIZING MAPS

Michel Haritopoulos, Hujun Yin and Nigel M. Allinson

Department of Electrical Engineering and Electronics
UMIST, Manchester, M60 1QD England
michelh@swift.ee.umist.ac.uk, {h.yin,n.allinson}@umist.ac.uk

ABSTRACT

This paper approaches the problem of image denoising from an Independent Component Analysis (ICA) perspective. Considering that the pixels intensity constituting the crude images represents the useful signal corrupted with noise, we show that, a nonlinear ICA-based approach can provide a satisfactory solution to the NonLinear Blind Source Separation problem (NLBSS). Self-Organizing Maps (SOMs) are well suited for performing this task, due to their nonlinear mapping property. Separation results obtained from test images demonstrate the feasibility of the proposed method.

1. INTRODUCTION

The Blind Source Separation (BSS) problem has become one of the main signal processing research areas, mainly due to its increasing application potential to many real world problems. A number of algorithms based on the Independent Component Analysis (ICA) theory have provided a solution to the BSS problems and have been successfully applied to a wide range of fields, covering diverse subjects such as biomedical signal processing, communications, underwater acoustics and image processing.

The BSS aims to recover a set of independent signals (also called sources) from a set of their observations (or mixtures) which form the sensor response. Unlike the Principal Component Analysis (PCA), ICA provides not only decorrelated but also statistically independent components from the observation variables set. The obtained solution is unique subject to some indeterminacies concerning the scale, sign and permutation of the recovered signals, only if the mixtures are linear combinations of the sources and eventually corrupted by some, usually additive, noise. But, in most real world cases, the signals of interest are mixed in a nonlinear manner, that makes necessary the application of other approaches to the source separation problem.

It has been proven that the solution to the NonLinear Blind Source Separation (NLBSS) problem is not unique [1]. However, there always exists at least one solution to the nonlinear case and by adding some additional constraints concerning the dimension of the source vector, the form of the mixing and the bounds of the densities of the independent components, the authors have shown that the NonLinear ICA (NLICA) solution is then unique up to a rotation.

Some neural network-based approaches to the NLBSS problem have also been proposed in the literature. Multilayer [2] or two-layer [3] perceptrons, as well as radial basis function (RBF) [4] and self-organizing maps (SOMs) [5] have been studied and applied to this difficult context. Further to a reminder about the ICA problem formulation and SOMs basic principles in the following two sections, we will present a SOM-based technique applied to the image denoising problem and the results we obtained after application to a test images set.

2. BSS THEORY BASICS

The BSS problem was first introduced by Hérault, Jutten and Ans [6], while the underlying mathematical tool providing a solution to this problem is ICA, which was first rigorously developed by P. Comon [7] as a generalization of the PCA technique. It aims to recover the unknown source signals from a set of their observations, in which they are mixed in a unknown manner.

2.1. ICA Formulation

Let us denote by $\mathbf{x}(t) = [x_1(t), ..., x_m(t)]^T \in \mathcal{X}$ a m-dimensional mixture vector from the observation space and by $\mathbf{s}(t) = [s_1(t), ..., s_n(t)]^T \in \mathcal{S}$ the unknown n-dimensional source vector from the source space at discrete time t, where the superscript T denotes the matrix transpose operation. Then, the generic BSS problem can be formulated as

$$\mathbf{x}(t) = \mathcal{F}[\mathbf{s}(t)] \quad (1)$$

where $\mathcal{F}$ is the unknown and generally nonlinear transformation of the source vector. If $\mathcal{F}$ is linear, then some assumptions are necessary in order to estimate its inverse. Precisely, the source vector components must be mutually inde-

This work is supported by a UK EPSRC grant GR/R 01460.

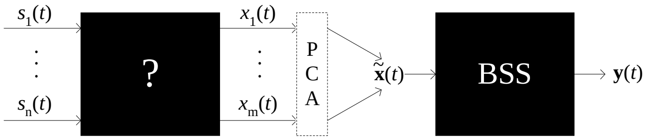


Fig. 1. Unknown mixing and separating structure for BSS.

pendent, $n \leq m$, the mixture is supposed stationary in time and at most one Gaussian source is allowed. Under these conditions and without any additional knowledge about either the sources or the mixtures, the linear ICA model yields to an estimate $\mathbf{y}(t) = \widehat{\mathbf{s}}(t)$ of the source vector:

$$\mathbf{y}(t) = \mathcal{F}^{-1}[\mathbf{x}(t)]. \tag{2}$$

In real world problems it is more likely for $\mathcal{F}$ to be nonlinear and there is always a noise term coming to perturb the noiseless model of eq. (1). This noise term $\nu(t)$ can be additive or multiplicative and of various distributions, correlated or not with the source signals. Thus, the complexity of the noisy NLICA model suggests the use of a flexible method which must be well adapted to the experimental context.

2.2. The Sphering Stage

A very common pre-processing step which often enhances convergence of BSS algorithms is "whitening" (or "sphering") of the mixtures $\mathbf{x}(t)$. It consists in a linear transformation of the centered observation vector, providing thus a new whitened vector $\widetilde{\mathbf{x}}$, i.e. a vector with decorrelated components and of unit variance. Many methods, as well as PCA, can be used for sphering the data, most of them being based on eigen-value or singular-value decomposition of the mixture covariance matrix, on triangular lower-upper decomposition or on triangular QR factorization. Note that, in the experimental part of the paper, we will use the PCA technique as a pre-processing step of the image data. Fig. 1 summarizes the mixing and the BSS processing systems.

3. SELF-ORGANIZING MAPS

Self-Organizing Maps (SOMs) [8], also known as Kohonen or topographic maps, are networks based on the principle of unsupervised learning. The latter can provide useful information about the input space, such as principal components, clusters or feature maps, since there is redundancy amongst the input data. Furthermore, the SOMs employ competition between their units and neighbourhood learning principle and transform the input space into a topology preserving nonlinear mapping.

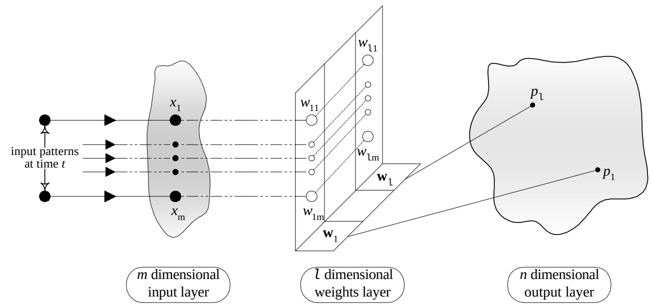


Fig. 2. Kohonen's SOM graphical scheme.

3.1. The SOM Algorithm

A SOM neural network contains three layers: a m-dimensional input layer $\mathcal{I}$, an intermediate layer $\mathcal{W}$ of arbitrary dimension denoted by l and a n-dimensional output layer $\mathcal{O}$. Each neuron j, $1 \leq j \leq l$ is connected to the input with a synaptic weight vector $\mathbf{w}_j = [w_{j1}, ..., w_{jm}]^T$. As it is illustrated by Fig. 2, given the input $\mathbf{x}$ at time t, the SOM network defines a best-matching (winner) neuron v:

$$\mathbf{v}(\mathbf{x}) = argmin_j \left\| \mathbf{x}(t) - \mathbf{w}_j \right\|, j = 1, ..., l \tag{3}$$

where $\|.\|$ denotes the Euclidean distance. Finally, the update of the synaptic weight vectors is made according to:

$$\mathbf{w}_j(t+1) = \mathbf{w}_j(t) + \eta(t) h_{j,\mathbf{v}(\mathbf{x})}(t) \left[\mathbf{x}(t) - \mathbf{w}_j(t)\right], \quad j = 1, ..., l \tag{4}$$

where $\eta(t)$ and $h_{j,\mathbf{v}(\mathbf{x})}$ designate the learning rate and the neighbourhood function centered around the winner, respectively.

If one deals with continuous signals, an interpolation between the winner and its neighbours is then necessary in order to make the map continuous. Geometric interpolation [9] uses orthogonal projections of the vector formed by the approximation to the exact input onto the one formed by the approximation to the second best-matching unit. Topological interpolation [10] instead, is based on a selection of the topological neighbours of the winner, which is an advantage over the geometrical method, unless there are topological defects on the chosen map. For our simulations, we used the component-wise linear interpolation method proposed in [11] as it has been proven sufficient for the NLBSS problem.

3.2. SOM and NLBSS of Continuous Sources

The earlier work on the application of SOM networks to the BSS amd NLBSS problem in [5], has proven that, under some constraints, this neural approach can provide a solution to the source separation problem. The farther the source probability density functions from the uniform one or the stronger the nonlinearities involved in the mixing process are, the more difficult a rigorous mapping of the input data to the nodes of the map is. This work has been extended in [11], where the authors studied the noise effect and the less sensors than sources case, particularly in presence of continuous sources.

Applying SOMs to the general NLBSS problem is straightforward especially for low dimensional maps. By matching the ICA observation space $\mathcal{X}$ and source space $\mathcal{S}$ to the SOM input layer $\mathcal{I}$ and output layer $\mathcal{O}$, respectively, it then remains to define the weights of the middle SOM layer and eventually perform an appropriate interpolation step to complete the separation task. The (continuous) source signals estimate vector $\mathbf{y}$ will then be the (interpolated) coordinates of the nodes $p_j, 1 \leq j \leq l$ of the map (Fig. 2). Note, that for special separating tasks such as image enhancement and denoising, a pre-processing step of the input data is necessary, as it will be outlined in the next session.

4. IMAGE DENOISING BY A 2-D SOM

Various methods ranging from wavelet shrinkage [12] to sparse code shrinkage [13] have already served to the image denoising problem, as alternatives to traditional median filters. Despite the drawbacks in the computational cost growing with the network complexity and quantization or interpolation errors [14], the data-driven and model-free approach together with the inherent nonlinear mapping from the input space to a usually low dimensional lattice that SOMs can offer, make this neural network a potential candidate for providing a solution to the NLICA problem applied to the image denoising research area.

Furthermore, most of the existing ICA techniques for BSS are robust against specific noise distributions or for low amplitude/noiseless model of eq. (1). Real world images are often corrupted, for example, by speckle, salt and pepper or quantization noise which can fit one or more distributions like Gaussian or Poisson and occur in an additive or multiplicative way. In many cases, nonlinear dependencies between two phenomena contributing to creating visual information appear correlated within the image. All these factors force many of the existing NLICA techniques to fail. In the remainder of this paper, we will propose a denoising technique based on SOMs networks which requires at least two image frames of the same scene.

4.1. Pre-processing and Definitions

4.1.1. The Pre-processing Step

Let us consider an image I of dimension $d_h \times d_w$ pixels (*height* $\times$ *width*). Attempt to denoise the whole digital image is translated by dealing with a random vector in a high dimensional space ($d_h \times d_w$ dimensions) which

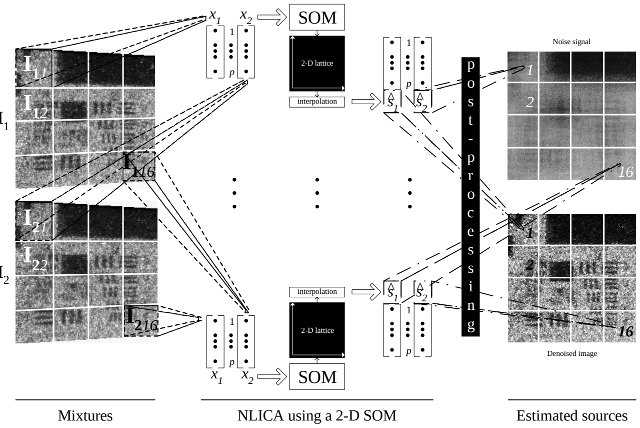


Fig. 3. Image pre-processing and neural separation.

would be computationally very expensive. One way to overcome this problem is "windowing". It partitions the image into distinct or sliding windows of wise-chosen dimensions $N_h \times N_w$ pixels. In our approach we opted for a distinct windowing approach in order to lower the computational cost at the expense of blocking artifacts which may arise in the treated image.

Hence, each one of the image frames is decomposed in $N = \frac{d_h}{N_h} \times \frac{d_w}{N_w}$ subimages I_j of dimension $p = N_h \times N_w$ pixels and rearranged in one dimensional vectors of length p. Finally, the local mean of the subimages has been subtracted and each I_j has been normalized. Also, various window sizes have been tested in order to determine the optimal one from a denoising point of view, as it will be pointed out later in this paper.

4.1.2. Sources and Mixtures

To apply a NLICA technique to this particular context by using a SOM network, we now have to determine the sources, mixtures and the "separability" property [15] which groups all the necessary assumptions for being able to apply an ICA technique.

Fig. 3 illustrates the SOM-based separation scheme proposed in this paper. It is based on a 2 mixtures - 2 sources approach, where the mixture vector $\mathbf{x}$ is composed by the corresponding subimages of the two available image shots, that we will denote by I_{1j} and I_{2j}, $1 \leq j \leq N$ after pre-processing as described in §4.1.1. As to the sources, we used a 2-D lattice for the SOM network in order to recover two separated sources $\mathbf{y}(t) = [\widehat{s}_1(t), \widehat{s}_2(t)]$ corresponding to the denoised subimage and noise respectively. After convergence of the SOM and interpolation of its output coordinates, we rearranged and repositioned the estimated vectors with respect to the whole image frame, obtaining thus a normalized version of the partially reconstructed denoised image and noise signal. Repeating this process to all of the N subimages I_{1j} and I_{2j}, one finally obtains the complete signal of interest, that is an enhanced version of the noisy image frames.

Verifying the validity of the assumptions necessary for applying a BSS scheme to these particular signals, is a very difficult task, due to our lack of prior knowledge concerning the real sources and the mixing process. Nevertheless, the experimental results will show that such an approach to the image denoising problem is quite feasible.

4.2. Indeterminacies

As we mentioned in §1, there are some intrinsic indeterminacies to the general BSS problem, whether it is a linear one or not. These become more undesirable, when no prior knowledge is available. In a totally blind context, the human factor seems to be important in order to inspect the visual information provided by the algorithms and eventually adjust the results or find an automation process that suits best its information quality requirements.

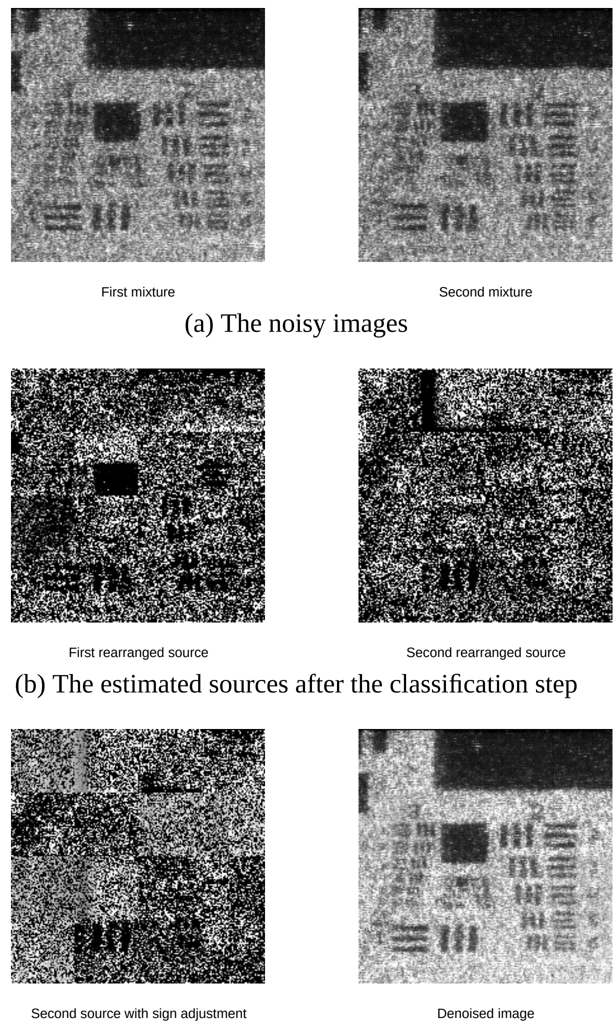
Indeed, as the whole image I is windowed, there is no guarantee that the entire set of the estimated subimages $\widehat{\mathbf{s}}_1$ and $\widehat{\mathbf{s}}_2$ will be of the same amplitude, scale and sign. This is more problematic, when the image is too large and p is chosen too small, unless a reliable technique is carried out to rearrange automatically the separated signals. Besides, the order of the estimated signals for each subimage does not necessarily remain the same; this means that a classification procedure after separation is needed. In the next section, we will propose a method to cope with this problem in our context.

5. EXPERIMENTAL RESULTS

The first images we used for our experiments were of $\lambda = 256$ grayscale levels quantized to one byte per pixel and of dimension 200×200 pixels. A total number of 24 consecutive frames were digitized from a CCD nightvision camera and they were focused in a test pattern containing numerals as well as horizontal and vertical bars.

5.1. SOM-based blind separation

For the windowing of the original image frames, different square window sizes have been tested and the total number of neurons on the 2-D output lattice was adjusted in consequence, as it is shown in Table 1. Fig. 4 (a) shows two non consecutive noisy frames issued from the CCD camera which, after pre-procesing as described in §4.1.1, served as entry to the 2-D SOM network with a square distinct windowing of 50 pixels.



(a) The noisy images

(b) The estimated sources after the classification step

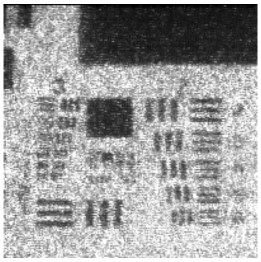
(c) Sign inversion and the mean denoised image

Fig. 4. The noisy images, the classification procedure and the denoising results.

Furthermore, in order to decide whether the SOM has reached a stable state, a pointer is kept to the previous values of the network weights over 1000 iterations and if the mean square error of their variation is smaller than a fixed threshold, for example $1e-6$, we consider that convergence is attained and the network is then stopped.

The first problem arising from this separation scheme concerns the order of the separated sources. In fact, nothing guarantees that the i^{th} and j^{th}, $1 \leq i, j \leq N$ separated subimages are ordered, i.e. that each one of the first and the second i^{th} and j^{th} estimated subimages belong to the first and the second global reconstructed images, respectively.

A solution to this problem can be provided by computing the correlation coefficient ρ between each one of the estimated normalized subimages and one of the correspond-



(a) Denoised image

(b) Median filter

Fig. 5. The SOM-based denoised image (a) and the best median filter (b) for two consecutive image frames.

ing normalized noisy subfames I_{1j} or I_{2j}, $1 \leq j \leq N$. Finally, the component of the estimated vector $\mathbf{y}$ with the greater correlation coefficient will belong to the first global estimated signal.

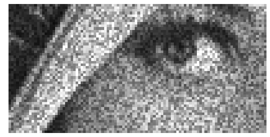
Table 1. Windowing size (in pixels) and SOM parameters.

$N_h = N_w$	10	20	40	50
p	100	400	1600	2500
N	400	100	25	16
$neurons$	18×18	35×35	70×70	87×87

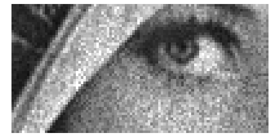
The results of such a classification procedure are illustrated by Fig. 4 (b). A total of 6 from 16 estimated source components have been rearranged according to their correlation coefficient with the first mixture of Fig. 4 (a). Note also that, after blind separation by the SOM network and classification , blocking artifacts due to the distinct windowing are hardly distinguishable because of the shapes geometry contained in the original noisy image frames.

5.2. Denoising results

For this image denoising problem the additive and multiplicative noise correlation models are investigated. The NLICA model of eq. (1) with an additive noise term, can be rewritten as $\mathbf{x}(t) = \mathcal{F}\left[\mathbf{s}(t) + \nu\,(t)\right]$ where $\nu(t)$ denotes the noise vector. The absence of prior knowledge concerning the noise contained in the images, led us to the following denoising procedure. We add (subtract) a slightly increasing quantity of the normalized estimated noise source to (from) the available noisy image frames and we average the resulting images. The optimal quantity α of the noise should be the one which maximizes the peak signal-to-noise ratio (PSNR); however, this requires additional knowledge concerning the noise-free image. Previous simulation experiments using this technique provided satisfactory results [16].



(a) Noisy image (noise variance 0.05)



(b) Denoised version (alpha=0.66)

Fig. 6. An artificial Lena mixture (a) and the optimal SOM-based denoised version (b).

Using the additive noise model to this images set, it yields to partially denoised images with a lot of information loss, that proves there exists a correlation between the original image and the noise. In this study, our work was then focused on the multiplicative noise model which is formulated by:

$$\mathbf{x}(t) = \mathcal{F}(\mathbf{s}(t)\,[1 + \nu(t)]). \tag{5}$$

Considering that the SOM network performed well and approximated a non linear mapping inhibiting the effects of the non linear mixing function $\mathcal{F}$, and taking the logarithm of both sides in the previous equation, one obtains:

$$\log(\mathbf{s}(t)) = \log(\mathbf{x}(t)) - \log\left[1 + \nu(t)\right]. \tag{6}$$

Here, the previously discussed α coefficient multiplies the last term of the right side of eq. (6). Exponentiation yields to the denoised versions of the available image frames, whose average is shown in Fig. 4 (c) ($\alpha = 0.85$).

Applying this technique, gives evidence of another problem regarding the separated sources and mentioned in §4.2. Some of the regions of interest in the denoised image are getting blurred with increasing the rate α of the estimated noise source. By empirically rearranging the sign of the estimated subimages which are concerned, this problem is overcame.

The overall proposed denoising method works also with consecutive noisy image frames, as it is illustrated by Fig.5 (a). In comparison, Fig.5 (b), illustrates the result of the best median filter applied to these two frames. One can remark that the background is part of the SOM-based estimated noise signal, that makes the remaining patterns contrast better. Note also, that using a very strong α coefficient makes gradually lose useful information within the image, that is, the estimated sources are still correlated one with the other.

Finally, employing individual neighbourhood for the SOM neurons as it is suggested in [11], could improve the results in the multiplicative noise model case. As an illustration, Fig.6 shows the results obtained by this method applied to a 50×100 pixels area of the well known Lena image, which has been artificially corrupted with multiplicative noise of variance 0.05. After application of SOM-based NLBSS to 8 pairs of subimages of size 25×25 pixels to the mixture of Fig.6 (a) and another noisy version of the same image, the automatic classification procedure described in §5.1 has been applied to the separated images and the optimal coefficient α was then computed, based in the maximization of the SNR. Hence, the denoised Lena image (Fig.6 (b)) enhances the SNR by $2.29db$.

6. DISCUSSION

In this paper, a SOM-based non linear blind source separation method has been applied to the image denoising problem. The presented results after its application to test images show that, even with only two available image frames, the denoising image has higher contrasts and carries out all the useful visual information contained within it. Furthermore, problems linked with the intrinsic indeterminacies of the BSS formulation, have been solved either automatically or empirically in a completely blind context.

However, this method is computationally expensive, so an optimization method dealing for example, with faster coding or parallelization, seems essential. This could allow the use of a "sliding" windows image partitioning which would smoother the denoising results, as it operates an average over the proposed image pixels values. Our current work is focused on the use of more than two mixtures and a method that could raise automatically all the indeterminacies discussed in this paper.

7. REFERENCES

[1] A. Hyvärinen and P. Pajunen, "Nonlinear independent component analysis: Existence and uniqueness results," *Neural Networks*, vol. 12, no. 3, pp. 429–439, 1999.

[2] T.-W. Lee, B.-U. Koehler, and R. Orglmeister, "Blind source separation of nonlinear mixing models," in *Proc. 1997 IEEE Workshop, Neural Networks for Signal Processing*, 1997, pp. 405–415.

[3] G. Burel, "Blind separation of sources: A nonlinear neural algorithm," *Neural Networks*, vol. 5, pp. 937–947, 1992.

[4] Y. Tan, J. Wang, and J. M. Zurada, "Nonlinear blind source separation using a radial basis function network," *IEEE Trans. Neural Networks*, vol. 12, no. 1, pp. 124–134, 2001.

[5] P. Pajunen, A. Hyvärinen, and J. Karhunen, "Nonlinear blind source separation by self-organizing maps," in *Progress in Neural Information Processing: Proc. ICONIP'96*, 1996, pp. 1207–1210.

[6] J. Hérault, C. Jutten, and B. Ans, "Détection de grandeurs primitives dans un message composite par une architecture de calcul neuromimétique en apprentissage non supervisé," in *Proc. GRETSI'85*, 1985, pp. 1017–1020.

[7] P. Comon, "Independent component analysis–a new concept?," *Signal Processing*, vol. 36, pp. 287–314, Apr. 1994, Special issue on Higher-Order Statistics.

[8] T. Kohonen, *Self-Organizing Maps*, Springer Series in Information Science no. 30. Springer, Berlin Heidelberg, 2nd edition, 1997.

[9] J. Göppert and W. Rosenstiel, "Topology preserving interpolation in self-organizing maps," in *Proc. NEURONIMES'93*, 1993, pp. 425–434.

[10] J. Göppert and W. Rosenstiel, "Topological interpolation in SOM by affine transformations," in *Proc. ESANN'95*, 1995.

[11] M. Herrmann and H. H. Yang, "Perspectives and limitations of self-organizing maps in blind separation of source signals," in *Progress in Neural Information Processing: Proc. ICONIP'96*, 1996, pp. 1211–1216.

[12] N. Weyrich and T. Warhola, "Wavelet shrinkage and generalized cross validation for image denoising," *IEEE Trans. Image Processing*, vol. 7, no. 1, pp. 82–90, 1998.

[13] P. Hoyer, "Independent component analysis in image denoising," M.S. thesis, Helsinki University of Technology, Espoo, 1999.

[14] H. H. Yang, S. Amari, and A. Cichocki, "Information back-propagation for blind separation of sources in non-linear mixture," in *Proc. IEEE ICNN'97*, 1997, pp. 2141–2146.

[15] X.-R. Cao and R. W. Liu, "General approach to blind source separation," *IEEE Trans. Signal Processing*, vol. 44, pp. 562–571, 1996.

[16] M. Haritopoulos, H. Yin, and N. Allinson, *Advances in Self-Organising Maps*, chapter "Nonlinear Blind Source Separation Using SOMs and Applciation to Image Denoising", pp. 275–282, Springer-Verlag, London, Nigel Allinson, Hujun Yin, Lesley Allinson and Jon Slack (Eds), June 2001.