A GENERAL THEORY OF CLOSED-FORM ESTIMATORS FOR BLIND SOURCE SEPARATION

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ABSTRACT

We present a general theory for the closed-form parametric estimation of the unitary mixing matrix after pre-whitening in the blind separation of two source signals from two noiseless instantaneous linear mixtures. The proposed methodology is based on the algebraic formalism of bicomplex numbers and is able to treat both real and complex valued mixtures indiscriminately. Existing analytic methods are found as particular cases of the exposed unifying formulation. Simulations in a variety of separation scenarios — even beyond the noiseless two-signal case — compare, assess and validate the methods studied.

1. INTRODUCTION

The present contribution addresses the blind separation of statistically independent source signals when instantaneous linear mixtures of the sources are observed. Second-order processing (pre-whitening) of the observed mixtures yields normalized uncorrelated components. In the noiseless case it is well known [1, 2, 3, 4] that the whitened sensor output $\mathbf{z} \in \mathbb{C}^r$ is related to the zero-mean unit-variance sources $\mathbf{x} \in \mathbb{C}^r$ through a unitary mixing transformation $Q \in \mathbb{C}^{r\times r}$:

$$\mathbf{z} = Q\mathbf{x}. \tag{1}$$

The general $r > 2$ environment can rely on the solution to the elementary $r = 2$ case by proceeding iteratively over the signal pairs [2]. We may thus focus on the latter scenario, in which the unitary transformation exhibits the general shape:

$$Q = \mathcal{Q}(\theta,\, \alpha) = \begin{bmatrix} \cos\theta & -\mathrm{e}^{-j\alpha}\sin\theta \\ \mathrm{e}^{j\alpha}\sin\theta & \cos\theta \end{bmatrix}. \tag{2}$$

Blind source separation (BSS) then reduces to a parametric estimation problem, the unknown parameters being the couple $(\theta,\, \alpha)$.

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The parameterization of matrix Q offers certain benefits. Closed-form (or analytic) expressions for the non-iterative estimation of the parameters of Q can be developed, based on the higher-order statistics of the observed signals. The main advantages of these closed-form estimators are their simplicity and mathematical tractability. In addition, it is usually possible to find further closed-form expressions that accurately predict the asymptotic (large-sample) performance of these methods.

In the case real-valued mixtures are observed, $\alpha = n\pi$, $n \in \mathbb{Z}$ (integers), and only θ is relevant to the source separation. Although apparently different closed-form solutions are found departing from a variety of disparate criteria, connections exist among them. Nulling the output 4th-order cross-cumulants [1] offers a non-uniform performance with the unknown parameter [5]. The approximate maximization of the observation truncated likelihood produced the analytic solution of [3] for sources with equal kurtosis, and was later extended (extended maximum likelihood, EML) to cater for a range of source distributions wider than originally designed [6]. Maximum likelihood reduces to a similar expression [7] (alternative EML, AEML) when the sources present opposite kurtosis values. In [8] another approximate maximizer of the truncated log-likelihood was found (approximate ML, AML). Maximization of the sum of squared output cumulants proves also fruitful at third (MaSSTOC) and fourth (MaSSFOC) orders [9, 10]. Some of these results were unified in [11]: a fourth-order estimation family (weighted estimator, WE) was developed from the linear weighting of the EML and AEML, and shown to originate MaSSFOC and AML as well at particular values of the weight coefficient.

By defining a new set of numbers (so-called bicomplex numbers) in accordance with the structure of matrix Q, some of these results were extended to complex-valued mixtures, evidencing a remarkable connection between the real and complex scenarios in the context of their analytical solutions [12]. This paper elaborates on the bicomplex-number theory to provide a generic framework for the analytic solutions to BSS. A common notation is developed to tackle both the real and the complex cases indiscriminately, thus allowing a unified derivation of closed-form estimators in both domains.

In the sequel, the cumulants of complex vector $\mathbf{z} = (z_1, \ldots, z_r)$ are defined as

$$\mathrm{Cum}^z_{i_1 i_2 i_3 \ldots} = \mathrm{Cum}[z_{i_1}, z^*_{i_2}, z_{i_3}, \ldots], \quad 1 \leqslant i_k \leqslant r, \tag{3}$$

symbol * denoting complex conjugation. The convention is adopted, in the the two-component case, that

$$\kappa^z_{n-p,\,p} = \mathrm{Cum}^z_{\underbrace{1\ldots1}_{n-p}\,\underbrace{2\ldots2}_{p}}. \tag{4}$$

2. SOLUTIONS IN REAL MIXTURES

The closed-form estimation of parameter θ in the real case is facilitated through the so-called complex centroids. These are complex-valued linear combinations of the whitened-sensor cumulants with the property that their phase is directly related to the parameter of interest. In fact, all the methods for real mixtures referred to in the previous section can be expressed in this form. The asymptotic variance of each method is linked to the quality of the source waveform recovery.

The *EML* estimator is based on the 4th-centroid [6]:

$$\xi_4 = (\kappa^z_{40} - 6\kappa^z_{22} + \kappa^z_{04}) + j4(\kappa^z_{31} - \kappa^z_{13}) = \gamma \mathrm{e}^{j4\theta}, \tag{5}$$

where $\gamma = (\kappa^x_{40} + \kappa^x_{04})$ and $j = \sqrt{-1}$. The source kurtosis sum (sks) can be estimated from the sensor output as

$$\gamma = \kappa^z_{40} + 2\kappa^z_{22} + \kappa^z_{04}. \tag{6}$$

Therefore, if $\gamma \neq 0$, angle θ may be determined as:

$$\hat{\theta}_{\mathrm{EML}} = \tfrac{1}{4}\angle(\xi_4 \gamma), \tag{7}$$

in which $\angle a$ represents the principal value of the argument of $a \in \mathbb{C}$.

AEML relies on [7]:

$$\xi_2 = (\kappa^z_{40} - \kappa^z_{04}) + j2(\kappa^z_{31} + \kappa^z_{13}) = \eta \mathrm{e}^{j2\theta}, \tag{8}$$

with $\eta = (\kappa^x_{40} - \kappa^x_{04})$. The value of the source kurtosis difference (skd) is not important (as long as it is different from zero), since it can only introduce a $\pm\pi/2$-rad bias which does not affect the source recovery.

AML and *MaSSFOC*, together with the two previous estimators, can all be derived from the weighted centroid [11]:

$$\xi_{\mathrm{WE}} = w\gamma\xi_4 + (1-w)\xi_2^2, \qquad w \in [0,\,1]. \tag{9}$$

Effectively, the AML is obtained for $w = 1/3$ whereas MaSSFOC results from $w = 1/2$. EML and AEML are deduced from $w = 1$ and $w = 0$, respectively. The asymptotic variance of the angle estimated from centroid (9) is given by [11]:

$$\sigma^2_{\mathrm{WE}} = \frac{\mathrm{E}\Big\{\big[w\gamma(x_1^3 x_2 - x_1 x_2^3) + (1-w)\eta(x_1^3 x_2 + x_1 x_2^3)\big]^2\Big\}}{T\big[w\gamma^2 + (1-w)\eta^2\big]^2}, \tag{10}$$

where T is the number of samples and $\mathrm{E}\{\cdot\}$ denotes the mathematical expectation. The value of the weight parameter yielding optimal finite-sample performance is also obtained in [11].

The performance of EML and AEML deteriorates for sks and skd near zero, respectively. This deterioration is avoided by the WE with any $w \in]0,\,1[$ [11]. It is somewhat striking to realize that other solutions based on optimality principles — such as AML (based on likelihood maximization) and MaSSFOC (based on contrast-function optimization) — originate from combinations of the EML and AEML while preventing their shortcomings.

Third-order estimator *MaSSTOC* [9] is derived from

$$\xi_3 = (\kappa^z_{30} - 3\kappa^z_{12}) + j(3\kappa^z_{21} - \kappa^z_{03}) = \gamma_3 \mathrm{e}^{j3\theta}, \tag{11}$$

in which $\gamma_3 = (\kappa^x_{30} - j\kappa^x_{03})$. In addition,

$$\gamma_3' = (\kappa^z_{30} + \kappa^z_{12}) - j(\kappa^z_{21} + \kappa^z_{03}) = \gamma_3 \mathrm{e}^{-j\theta}. \tag{12}$$

Hence, $\xi_3(\gamma_3')^*$ estimates $|\gamma_3|^2 \mathrm{e}^{j4\theta}$, from which θ can easily be determined when at least one of the sources is asymmetrically distributed. We calculate its asymptotic variance as:

$$\sigma^2_{\mathrm{MaSSTOC}} = \frac{\mu^x_{40}(\mu^x_{30})^2 + \mu^x_{04}(\mu^x_{03})^2 - 2(\mu^x_{30}\mu^x_{03})^2}{T[(\mu^x_{30})^2 + (\mu^x_{03})^2]^2}, \tag{13}$$

with $\mu^x_{mn} = \mathrm{E}\{x_1^m x_2^n\}$.

3. A GENERAL THEORY BASED ON BICOMPLEX-NUMBER REPRESENTATION

The complex centroids are useful in estimating parameter θ when real mixtures are observed. However, these representations cannot cope with complex-valued mixtures which arise due to parameter α in matrix Q and/or to complex-amplitude sources. To allow the complex mixture identification, a special class of numbers is introduced as follows.

3.1. Bicomplex Numbers

A bicomplex number $\bar{x} = a + \jmath b,\ a,\, b \in \mathbb{C}$, is defined from the first column of orthogonal matrix $U = \left[\begin{smallmatrix} a & -b^* \\ b & a^* \end{smallmatrix}\right]$. Terms $a = \mathbb{R}\mathrm{e}(\bar{x})$ and $b = \mathbb{I}\mathrm{m}(\bar{x})$ are the *breal* and *bimaginary* parts of $\bar{x}$, respectively, which are to be distinguished from the commonplace real [$\Re(a) \in \mathbb{R}$] and imaginary [$\Im(a) \in \mathbb{R}$] parts of $a \in \mathbb{C}$. Symbol $\jmath$ is named *bimaginary unit.*

The product of two bicomplex numbers $\bar{x}_1 = a_1 + \jmath b_1$ and $\bar{x}_2 = a_2 + \jmath b_2$ is defined by taking into account the first column of the product of the corresponding orthogonal matrices:

$$\bar{x}_1\bar{x}_2 = (a_1a_2 - b_1^*b_2) + \jmath(b_1a_2 + a_1^*b_2). \tag{14}$$

In this manner an isomorphism is created between the set of orthogonal matrices under usual matrix product and the set of bicomplex numbers under the above product operation.

A bicomplex number is formed by a pair of complex quantities. Also, note that $\jmath^2 = j^2 = -1$. However, it must be stressed that $\jmath$ and j represent distinct algebraic elements. In particular, $r + ji \neq r + \jmath i$, with $r,\, i \in \mathbb{R}$. The first number is actually $(r + ji) + \jmath 0$, whereas the second can be written as $(r + j0) + \jmath(i + j0)$. As far as BSS is concerned, both are essentially different. In the former case the associated orthogonal matrix is always non-mixing, but in the latter case it would generally not be so. Hence, $a \in \mathbb{C}$ is uniquely represented by the breal part of $\bar{x} = a + \jmath 0$.

The bicomplex number associated with a unitary transformation like (2) is called bicomplex exponential:

$$\mathrm{e}_{\alpha}^{\jmath\theta} = \cos\theta + \jmath \mathrm{e}^{j\alpha}\sin\theta, \tag{15}$$

Remark that the bicomplex product is generally non-commutative: $\bar{x}_1\bar{x}_2 \neq \bar{x}_2\bar{x}_1$. For instance,

$$\jmath \mathrm{e}_{-\alpha}^{\jmath\theta} = \mathrm{e}_{\alpha}^{\jmath\theta}\,\jmath. \tag{16}$$

Similarly, the product of a complex and a bicomplex number does not commute, but the product of a real and a bicomplex number always does.

The existing isomorphism between bicomplex numbers and (2×2) orthogonal matrices enables the straightforward definition of operators such as conjugation and modulus:

$$\bar{x}^* = \mathbb{R}\mathrm{e}(\bar{x})^* - \jmath \mathbb{I}\mathrm{m}(\bar{x}) \tag{17}$$

$$|\bar{x}| = (\bar{x}\bar{x}^*)^{\frac{1}{2}} = (\bar{x}^*\bar{x})^{\frac{1}{2}} = \left(|\mathbb{R}\mathrm{e}(\bar{x})|^2 + |\mathbb{I}\mathrm{m}(\bar{x})|^2\right)^{\frac{1}{2}}. \tag{18}$$

3.2. A Family of Bicomplex Centroids

Bicomplex centroids can be defined as particular bicomplex weighted sums of the whitened vector (perhaps complex) statistics. These linear combinations are such that their 'phase' is a function of the unknown parameters.

Theorem 1. *Let $\bar{\xi}_{n,m}^z$ be the following bicomplex weighted sum of pairwise βth-order cumulants of the components of* $\mathbf{z}$*, with $n,\, m \in \mathbb{N}$ (non-negative integers) and $\beta = n + 2m$:*

$$\bar{\xi}_{n,m}^z \triangleq \sum_{p=0}^{n} \binom{n}{p} [(-1)^m \jmath]^p \sum_{q=0}^{m} \binom{m}{q} \kappa_{\beta-(p+2q),\,p+2q}^z. \tag{19}$$

If $\mathbf{z} = \mathcal{Q}(\theta,\, \alpha)\,\mathbf{x}$, with $\mathbf{x}$ made up of independent components, then:

$$\bar{\xi}_{n,m}^z = \mathrm{e}_{(-1)^{n-1}\alpha}^{\jmath(-1)^m n\theta}\,\bar{\xi}_{n,m}^x \tag{20}$$

where, according to (19)*,*

$$\bar{\xi}_{n,m}^x = \kappa_{\beta,0}^x + [(-1)^m \jmath]^n \kappa_{0,\beta}^x. \tag{21}$$

Sketch of the proof. The proof is essentially based on the multilinearity property of cumulants [13], the source statistical independence assumption, the unitary nature of matrix Q, trigonometric identities and certain algebraic simplifications. The fact that bicomplex product is non-commutative must also be borne in mind [e.g., eqn. (16)]. ∎

The above result is not practical by itself, since by definition the source cumulants are unknown in a blind situation. Nevertheless, the following corollary surmounts the lack of knowledge on the source statistics in many cases.

Corollary 2. Let

$$\bar{\Xi}_{n,m,k}^z = \bar{\xi}_{n,m}^z (\bar{\xi}_{n-2k,m+k}^z)^*, \tag{22}$$

with $k \in \mathbb{N}$, such that $(n-2k) \geqslant 0$. If $[k(n+1)] \bmod 2 = 0$ then:

$$\bar{\Xi}_{n,m,k}^z = |\bar{\xi}_{n,m}^x|^2 \mathrm{e}_{(-1)^{n-1}\alpha}^{\jmath(-1)^m\delta\theta} \tag{23}$$

where $\delta = n - (-1)^k(n-2k)$.

Proof. When $(-1)^{k(n+1)} = 1$, source centroid $\bar{\xi}_{n-2k,m+k}^x$ equals $\bar{\xi}_{n,m}^x$, so from eqns. (17)–(18) and (22), result (23) follows. ∎

3.3. Blind Parameter Estimation

3.3.1. General result

With the conditions of Corollary 2, and if at least one of the source βth-order marginal cumulants is different from zero, the unknown parameters can be estimated from the whitened-vector statistics via:

$$\hat{\theta} = (-1)^m \delta^{-1} \angle\Big(\mathbb{R}\mathrm{e}(\bar{\Xi}_{n,m,k}^z) + j\big|\mathbb{I}\mathrm{m}(\bar{\Xi}_{n,m,k}^z)\big|\Big) \tag{24a}$$

$$\hat{\alpha} = (-1)^{n-1} \angle\, \mathbb{I}\mathrm{m}(\bar{\Xi}_{n,m,k}^z), \tag{24b}$$

3.3.2. Identifiability constraints

The values of δ producing a valid source separation are limited. The source waveforms are preserved when solutions $(\hat{\theta},\, \hat{\alpha})$ are of the form:

$$(\hat{\theta},\, \hat{\alpha}) = \big((-1)^u\theta + \tfrac{v\pi}{2},\, \alpha + u\pi\big), \quad u,\, v \in \mathbb{Z}. \tag{25}$$

In particular, only phase shifts integer multiples of $\pi/2$ are permitted in the estimation of θ. Since $\mathrm{e}_{\omega}^{\jmath\delta(\theta+2\pi v/\delta)} = \mathrm{e}_{\omega}^{\jmath\delta\theta}$, $\forall \omega \in \mathbb{R}$, coefficient δ must be restricted to the set $\{2,\, 4\}$.

On the one hand, $\delta = 2$ is generated from $n = 2$ and $k = 1$, with cumulant orders $\beta = 4, 6, 8, \ldots$. On the other hand, $\delta = 4$ is generated from $k = 1$, $n = 3$ ($\beta = 3, 5, 7, \ldots$) and $k = 2, \forall n \geqslant 4$ ($\beta \geqslant 4$).

As in the WE, centroids at different cumulant orders can be combined in a bid to increase the estimation accuracy and the robustness against noise or impulsive interference [8, 11].

3.3.3. *Particular cases*

Some particular cases of the triplet (n, m, k) are contemplated below.

Case $(4, 0, 2)$:

$$\begin{aligned} \bar{\xi}^z_{4,0} &= (\kappa^z_{40} - 6\kappa^z_{22} + \kappa^z_{04}) + \jmath 4(\kappa^z_{31} - \kappa^z_{13}) \\ \bar{\xi}^z_{0,2} &= \kappa^z_{40} + 2\kappa^z_{22} + \kappa^z_{04} \\ \bar{\Xi}^z_{4,0,2} &= \gamma^2 \mathrm{e}^{\jmath 4\theta}_{-\alpha}. \end{aligned} \tag{26}$$

These equations correspond to the EML estimator (5)–(7), which is hence naturally extended to the complex-mixture domain [11].

Case $(2, 1, 0)$ requires an *ad hoc* subtle re-definition of $\bar{\Xi}^z$:

$$\begin{aligned} \bar{\xi}^z_{2,1} &= (\kappa^z_{40} - \kappa^z_{04}) - \jmath 2(\kappa^z_{31} + \kappa^z_{13}) \\ \bar{\Xi}^z_{2,1,0} &\triangleq (\bar{\xi}^{z\,*}_{2,1})^2 = \eta^2 \mathrm{e}^{\jmath 4\theta}_{-\alpha} \end{aligned} \tag{27}$$

which extends the AEML centroid (8) [11].

Cases $(4, 0, 2)$ and $(2, 1, 0)$ can also be combined as in the WE (9), giving rise to the hybrid centroid:

$$\bar{\Xi}^z_{\mathrm{WE}} = w\,\bar{\Xi}^z_{4,0,2} + (1-w)\,\bar{\Xi}^z_{2,1,0}, \quad 0 \leqslant w \leqslant 1, \tag{28}$$

which estimates $[w\gamma^2 + (1-w)\eta^2]\mathrm{e}^{\jmath 4\theta}_{-\alpha}$. As in the real case, for $w \in]0, 1[$ the WE does not experience the performance degradation of EML and AEML when the sks and the skd tend to zero, respectively [11]. At $w = 1/3$ and $w = 1/2$, the corresponding methods become, resp., the complex extensions of the AML and MaSSFOC estimators.

Case $(3, 0, 1)$:

$$\begin{aligned} \bar{\xi}^z_{3,0} &= (\kappa^z_{30} - 3\kappa^z_{12}) + \jmath(3\kappa^z_{21} - \kappa^z_{03}) \\ \bar{\xi}^z_{1,1} &= (\kappa^z_{30} + \kappa^z_{12}) - \jmath(\kappa^z_{21} + \kappa^z_{03}) \\ \bar{\Xi}^z_{3,0,1} &= |\gamma_3|^2 \mathrm{e}^{\jmath 4\theta}_{\alpha}. \end{aligned} \tag{29}$$

This is the generalized version of the MaSSTOC estimator (11)–(12).

When real-valued mixtures are treated, eqns. (24) on the above bicomplex centroids are tantamount to the argument function on the corresponding complex centroids summarized in Section 2. Consequently, the presented estimators can be applied regardless of the mixture type.

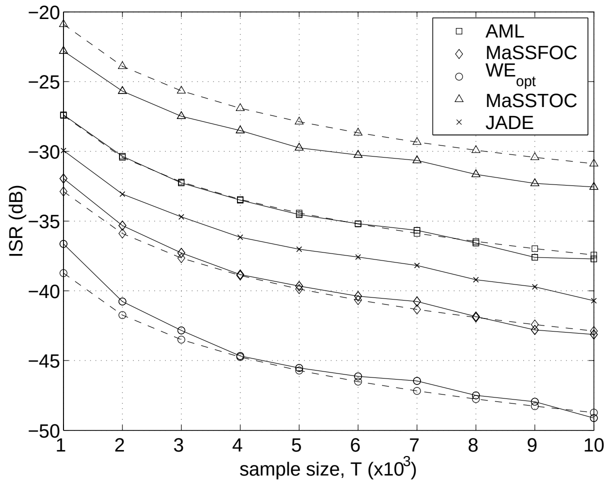

Fig. 1. ISR vs. sample size. Binary–Rayleigh sources, $\theta = 30^{\mathrm{o}}$, ν independent MC runs, with $\nu T = 5 \times 10^6$. Solid lines: average empirical values. Dashed lines: asymptotic variances (10) (WE) and (13) (MaSSTOC).

4. EXPERIMENTAL RESULTS

A number of computer experiments illustrate the above theoretical results and evaluate the proposed methods in a variety of scenarios. Closed-form separations are obtained with the bicomplex-centroid formalism developed in Section 3 without taking into account the type of mixture (real or complex) being observed. For the sake of comparison, a well established BSS procedure not based on closed-form estimation, JADE [14], is also considered.

Fitness of asymptotic analysis

The first experiment tests the theoretical asymptotic results of Section 2. Source realizations are composed of i.i.d. samples drawn from (symmetric) binary and Rayleigh distributions. The performance of separations carried out on real-valued orthogonal mixtures is averaged over several independent Monte Carlo (MC) runs. The interference to signal ratio (ISR) [4] is used as an objective performance index. In the real case, the ISR approximates the estimated-angle variance for $\hat{\theta}$ near effective separation solutions. Fig. 1 shows that the WE asymptotic variance (10) accurately approximates the empirical outcomes for the tested values of the weight coefficient w, even for relatively short data records. MaSSTOC asymptotic variance (13) appears slightly pessimistic. In this simulation, both the optimal WE ($w_{\mathrm{opt}} = 0.6319$) [11] and MaSSFOC outperform JADE; in fact, the optimal WE is about 5 times as efficient [15] as JADE.

Performance variation with angular parameters

The influence of the actual values of the unitary-mixing parameters is assessed next. First, the source effect on the observed complex cumulants is neutralized by considering sources with real-valued uniform and exponential distribu-

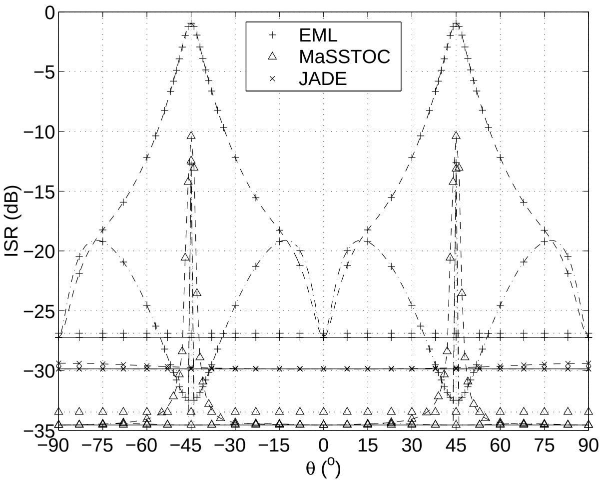

Fig. 2. ISR vs. unitary mixing matrix parameters. Uniform–exponential sources, $T = 5 \times 10^3$ samples, 200 MC runs. Solid, dashed and dash-dotted lines: $\alpha = 0^{\mathrm{o}},\ 45^{\mathrm{o}},\ 90^{\mathrm{o}}$ (resp.). Dotted lines: theoretical variances (10) (EML) and (13) (MaSSTOC).

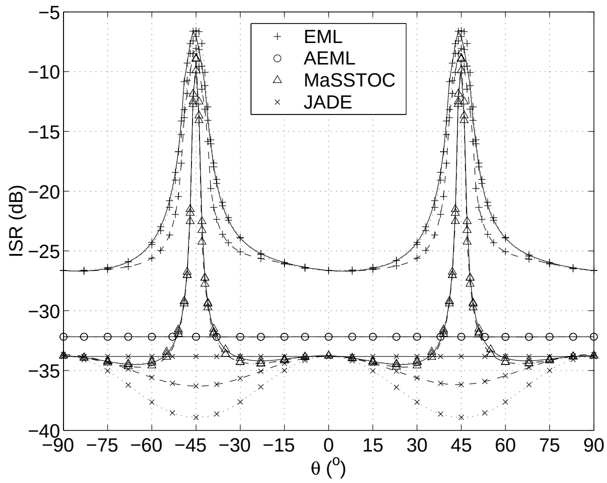

Fig. 3. ISR vs. unitary mixing matrix parameters. 4-QAM sources with kurtosis $(-1,\ 2)$, $T = 5 \times 10^3$ samples, 200 MC runs. Solid, dashed and dotted lines: $\alpha = 0^{\mathrm{o}},\ 45^{\mathrm{o}},\ 90^{\mathrm{o}}$ (resp.).

tion. At each $(\theta,\ \alpha)$ value, separations are obtained from the corresponding mixtures of a fixed (but otherwise randomly selected) set of source realizations. Fig. 2 shows the average ISR curves for EML, MaSSTOC and JADE as a function of θ, for three different values of α. EML performance suffers considerable variations with θ when $\alpha \neq 0$. Variations are less severe for MaSSTOC, whose performance is flat and improves JADE by about 4 dB in most θ-range, but worsens near $\theta = \pm 45^{\mathrm{o}}$ for $\alpha \neq 0$. AEML results (not shown for clarity) lie just below JADE's in this example, and do not depend on the parameters of the unitary mixing matrix.

Fig. 3 presents analogous results with complex-valued sources (4-QAM signals with normalized kurtosis -1 and 2). Again, AEML's performance is uniform over $(\theta,\ \alpha)$. EML and MaSSTOC show a more regular behaviour than in the previous simulation, although their performance deteriorates around $\theta = \pm 45^{\mathrm{o}}$.

The performance degradation experienced by EML and MaSSTOC may stem from the fact that $\mathbb{Im}(\mathrm{e}^{\jmath 4\theta}_{-\alpha}) = 0$ when $\theta = v\pi/4$, $v \in \mathbb{Z}$, which hinders the estimation of α from their respective centroids [eqns. (26) and (29)] at such values of θ. As pointed out in [11, Sec. 3.3], the estimation of α is immaterial for v even. For v odd, however, inaccurate estimates of α do have an impact on the separation quality.

More than two sources in noise

Finally, the methods' performance is evaluated when more than two mixtures of more than two sources are observed in the presence of noise. To deal with this general scenario, the pairwise approach of [2] is adopted: analytic two-signal estimators are applied over the whitened component pairs successively, over a number of sweeps. Complex-valued mixing matrix entries are made up of independent real and imaginary parts uniformly distributed in the interval $[-1,\ 1]$. The signal-to-noise ratio (SNR), that we define as the power due to sources over the power due to noise for a given observed signal, is chosen to be equal at all sensors. Pre-whitening is achieved via the singular value decomposition of the observation sample matrix. Fig. 4 shows average ISR results for three observed mixtures of three 4-QAM sources with normalized kurtosis values $(-1,\ 1,\ 1)$ embedded in additive noise with complex Gaussian distribution. Even though the analytic estimators were developed in the noiseless two-signal BSS scenario, they also prove successful in separating more than two sources in noise. Only EML and AEML fail even at positive SNR because source pairs with zero sks or skd exist. The weighted estimators (AML, MaSSFOC) and MaSSTOC follow closely JADE's trend. MaSSTOC even improves JADE in the low SNR range $[-10,\ 0]$.

Similar comments hold in the experiment of Fig. 5, in which impulsive interference is simulated by means of 4-QAM noise signals with normalized kurtosis $(3,\ 3,\ 3)$. In this occasion AML, MaSSTOC and MaSSFOC remain even closer to JADE. To start separating the sources effectively, the methods need about 10 dB higher SNR than in the Gaussian noise experiment, since the non-null higher-order statistics of the interference now affect the higher-order stage of the separation.

5. CONCLUSIONS

A novel formulation for the closed-form estimation of the unitary mixing matrix parameters after the pre-whitening stage in two-signal instantaneous linear BSS has been put

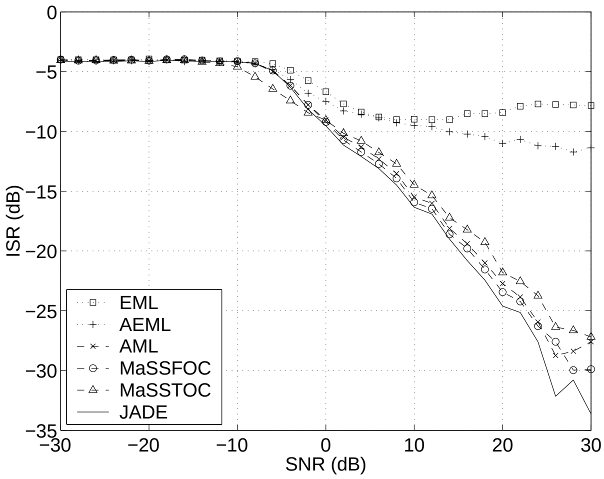

Fig. 4. ISR vs. SNR. 4-QAM sources with kurtosis $(-1,\,1,\,1)$, complex Gaussian noise, mixing matrix elements with independent real and imaginary parts uniformly distributed in $[-1,\,1]$, $T = 5 \times 10^3$ samples, 10^3 MC runs.

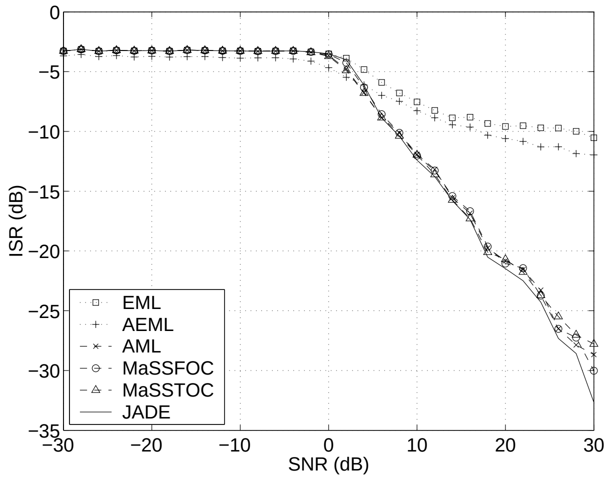

Fig. 5. ISR vs. SNR. 4-QAM sources with kurtosis $(-1,\,1,\,1)$, 4-QAM noise with kurtosis $(3,\,3,\,3)$, mixing matrix elements with independent real and imaginary parts uniformly distributed in $[-1,\,1]$, $T = 5 \times 10^3$ samples, 10^3 MC runs.

forward. The approach emerges from the fusion of the notion of centroid — a linear combination of the whitened-vector higher-order cumulants retaining information about the unknown parameters — and the algebraic formalism of the bicomplex numbers — which are isomorphic to the matrix set to be identified. The resulting estimators are applicable regardless of the mixture type, either real or complex valued. The developed theory unifies the formulation of existing analytic methods (EML, AEML, AML, MaSSFOC, WE, MaSSTOC) and accomplishes their natural extension to the complex-mixture scenario, which is of relevance in disciplines such as digital communications and seismic exploration. Simulations have shown that the estimators are also useful in the presence of noise and, by operating pairwise, can successfully tackle the general BSS scenario of more than two signals.

The observed non-uniform behaviour of EML and MaSSTOC with the unknown parameters is not yet fully understood, and deserves further investigation. The theoretical performance analysis of the closed-form estimators in the complex domain could shed some light on this issue.

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