# Sequential Least Squares

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Let

$$\rho(\theta, t) = (1-\gamma)\Big(\sum_{s=0}^{\infty} \gamma^s e^2(\theta, s)\Big) \tag{1}$$

where

$$e^2(\theta, s) = \frac{1}{2}(y_s - x_s'\theta)^2 \tag{2}$$

Note

$$\frac{\partial e^2(\theta, s)}{\partial \theta} = x_s y_s - x_s x_s' \theta \tag{3}$$

and

$$\frac{\partial \rho(\theta, t)}{\partial \theta} = (1-\gamma)\Big(\sum_{s=0}^{\infty} x_s y_s\Big) - (1-\gamma)\Big(\sum_{s=0}^{\infty} x_s x_s'\Big)\theta \tag{4}$$

$$= f_{xy}(t) - f_{xx}(t)\theta \tag{5}$$

where

$$f_{xx}(t) = (1-\gamma)\Big(\sum_{s=0}^{\infty} \gamma^s x_s x_s'\Big) \tag{6}$$

$$f_{xy}(t) = (1-\gamma)\Big(\sum_{s=0}^{\infty} \gamma^s x_s y_s\Big) \tag{7}$$

$$\tag{8}$$

The minimum or $\rho(\theta, t)$ with respect to $\theta$ is achieved for

$$\theta_t = f_{xx}^{-1}(t) f_{xy}(t) \tag{9}$$

Note the $f_{xx}$ matrix and the $f_{xy}$ vector can be computed in a recursive manner

$$f_{xx}(t) = (1-\gamma)x_t x_t' + \gamma f_{xx}(t-1) \tag{10}$$

$$f_{xy}(t) = (1-\gamma)x_t y_t + \gamma f_{xy}(t-1) \tag{11}$$

$$\tag{12}$$

Moreover, the inverse matrix $f_{xx}^{-1}(t)$ can also be computed in a recursive manner. Using the matrix inversion theorem

$$f_{xx}^{-1}(t) = \frac{1}{\gamma}\Big(f_{xx}^{-1}(t-1) - \frac{\alpha}{1+\alpha x_t' z_t} z_t z_t'\Big) \tag{13}$$

where

$$\alpha = \frac{1-\gamma}{\gamma} \tag{14}$$

$$z_t = f_{xx}^{-1}(t-1) x_t \tag{15}$$

# 1 Appendix: Example Code

```
% Example Matlab Code for Sequential Least Squares
% Javier R. Movellan. Feb 2010
clear
```

```
p = 10; % The dimensionality of the imput
T = 500; % The number of time steps
gamma = 0.99;
alpha = (1-gamma)/gamma;

trueTheta = (1:p)';
x = randn(p,T);
y = x'*trueTheta;



theta= zeros(p,T); % our running estimate of theta;

fxxinv = eye(p); % Initial value of the fxxInv matrix
fxy = zeros(p,1); % Initial value of the fxy vector
theta(:,1) = fxxinv*fxy;


for t=2:T
  xt = x(:,t);
  zt = fxxinv*xt;
  fxy= (1-gamma)*xt*y(t)+ gamma*fxy;
  fxxinv = (fxxinv - (alpha/(1+ alpha*xt'*zt))*zt*zt')/gamma;
  theta(:,t) = fxxinv*fxy;
end

plot(theta')
xlabel('Trial Number');
ylabel( '\theta');
```