Let positive integers \( s_1 \) and \( s_2 \) denote the “speeds” of Iterator 1 and 2, respectively, where speed is the number of nodes that each iterator traverses in one timestep. Let non-negative integer \( l \) denote the number of nodes in the linked list prior to the start of the loop. Finally, let positive integer \( k \) denote the number of nodes in the loop.

Here we show that the two iterators must “intersect” at the same node inside the loop after a finite number of timesteps. An intersection is defined to occur when the “distance” of Iterator 1 from the start of the loop is equal to the “distance” of Iterator 2 from the start of the loop at the same moment in time. The distance is a number in \( \{0, \ldots, k-1\} \) that specifies which node in the loop (starting at 0) the iterator currently points to. Let \( m \) be the smallest non-negative integer such that \( ms_1 \geq l \) and \( ms_2 \geq l \). In other words, \( m \) is the number of timesteps that Iterator 1 and 2 must iterate so that they have both entered the loop. Then \( ms_1 - l \mod k \) (or \( ms_2 - l \mod k \)) is the distance of Iterator 1 (or Iterator 2) from the start of the loop after \( m \) timesteps.

We wish to find a number of additional timesteps \( t \geq 0 \) (after the initial \( m \) timesteps) such that the two iterators intersect in the loop. More precisely, we wish to find a \( t \) that satisfies the congruency

\[
ms_1 - l + ts_1 \equiv ms_2 - l + ts_2 \mod k \quad (1)
\]

\[
m(s_1 - s_2) + t(s_1 - s_2) \equiv 0 \mod k \quad (2)
\]

\[
(m + t)(s_1 - s_2) \equiv 0 \mod k \quad (3)
\]

If \( s_1 = s_2 \) (i.e., the iterators travel at the same speed), then this congruency is satisfied for all values of \( t \). Otherwise \( (s_1 \neq s_2) \), this congruency is satisfied whenever \( m + t = nk \) for any integer (of our choosing) \( n \geq 0 \) because then \( (m + t)(s_1 - s_2) \) is an exact multiple of \( k \). In order to guarantee non-negativity of \( t \), we choose \( n \) to be the smallest integer such that \( nk \geq m \). Then, a suitable \( t \) can be found by solving

\[
m + t = nk \quad (4)
\]

\[
t = nk - m \quad (5)
\]

Since we have found a \( t \geq 0 \) at which both iterators intersect, and since \( t \) was the number of additional timesteps after the initial \( m \) timesteps required to enter the loop, we conclude that the two iterators will intersect after a total of \( t + m = nk - m + m = nk \) timesteps, where \( n \) is the smallest non-negative integer such that \( nk - m \geq 0 \).